Topology (Back paper)
(BMath-2nd year, 2025)

Instructions: Total time 2 Hours. All the problems are compulsory, maximum marks are 60. Use terminologies, notations and results as covered in the course, no need to prove such results. If you wish to use a problem given in an assignment or homework or any other such, please supply its full solution.

1. Let $A = \{(0,x) \in \mathbb{R}^2 | x \text{ is irrational and } x < 1\}$ and B denote the deleted comb space:
$$B = (([0,1] \times \{0\}) \cup (K \times [0,1])) \cup \{p\}),$$
here $K = \{1/n | n \in \mathbb{Z},\ n \geq 1\}$ and p denotes the point $(0,1) \in \mathbb{R}^2$. Let $X = A \cup B \subset \mathbb{R}^2$ have the subspace topology from $\mathbb{R}^2$. (i) Prove that X is connected. (ii) Is X path connected? If not, how many path components does X have? Explain. (5+10)

2. Consider the quotient space X obtained from the Möbius band by identifying the boundary circle to a point. Then X is homeomorphic to which of the following spaces (i) Klein bottle (ii) $\mathbb{R}P^2$ (iii) $\mathbb{R}P^3$ (iv) S^2? Justify your answer. (10)

3. Let $n \geq 1$ and $f : S^n \to \mathbb{R}$ be continuous. Prove that for some $x \in S^n$, $f(x) = f(-x)$. (10)

4. Let X and Y be topological spaces, $p : X \to Y$ be a continuous map. Suppose there is a continuous map $f : Y \to X$ such that $p \circ f = \mathrm{Id}_Y$, the identity map on Y. Prove that p is a quotient map. (10)

5. Let X be a normal topological space and A a closed subset of X. Prove that the quotient space X/A is normal. (10)

6. True/False: Continuous image of a normal space is normal. Justify your answer. (5)